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GENERATING CYCLES REFLECTED IN A CENTURY OF PRICES

SUMMARY

Cycles of approximately eight years in the yield per acre of British crops have probably recurred in a continuous series during the last one hundred and sixty years. In consequence of the law of demand, these crop cycles have generated corresponding cycles in the prices of food and of organic raw materials. In conformity with the law of competitive price, the cycles in the prices of food and of raw materials should have been followed by corresponding cycles in the prices of manufactured commodities. As a matter of fact, the analysis of Sauerbeck's index numbers of general wholesale prices reveals real cycles of approximately eight years in which the originating, generating crop cycles are reflected throughout the century for which the Sauerbeck index numbers have been computed.

The principal results of the investigation appear in the graphs of Figures 5 and 6.

In a recent paper [1] I drew a distinction between generating economic cycles and derived economic cycles. Generating economic cycles were described as economic cycles that have their origin in non-economic causes and become the originating source of derived economic cycles. A careful scrutiny of American agricultural statistics revealed the existence of an eight-year gen-

[1] "Generating Cycles of Products and Prices," Quarterly Journal of Economics, February, 1921, pp. 215–239.

erating cycle in the yield per acre of the leading American crops, and this generating cycle of products was found to originate a derived cycle of agricultural prices. American manufactures, according to the census of 1900, obtained about 80 per cent of their raw materials from the farms, and as the prices of manufactured commodities tend to adjust themselves to the cost of production, it was argued that the eight-year cycles in the prices of farm products tended to induce derivative cycles in the prices of manufactured commodities. The major features of economic cycles were regarded as being traceable to three primary laws:

(1) the law of the generating cycle of raw materials, which is due to a non-economic cause;
(2) the law of demand for raw materials, in consequence of which the generating cycle of products originates a derived cycle of prices for raw materials; and
(3) the law of competitive price, according to which the prices of finished goods in an open market tend to correspond with the cost of production.

The present paper carries the inquiry a stage further. An analysis is made of the history of prices in Great Britain, for a century, and the results are considered with reference to their dependence upon generating agricultural cycles and with regard to their bearing upon the economic theory of cycles.

I

Data and Method

The data for the first part of the investigation are the Sauerbeck index numbers of general wholesale prices for the interval between the Napoleonic wars and the Great War. This record from 1818 to 1913, which is

a summary description of economic history during a century of unparalleled development between two world-wide catastrophes, supplies unique material for an inductive quest of economic regularities.

The method used in the inquiry is Fourier's Theorem as it has been developed for statistical purposes by Professor Schuster and Professor Turner.

The Savilian Professor of Astronomy at Oxford, Professor Turner, has said that "apart from the planetary motions periodicities in nature are seldom clearcut."[1] As we shall attempt to establish certain economic cycles and to trace their cause to periodicities in nature, the part of wisdom would seem to be to profit by the experience of natural scientists who have dealt with the problem of isolating natural periodicities.

It is well known that Fourier's celebrated theorem

$$y = A_0 + a_1 \cos kt + a_2 \cos 2kt + \ldots$$
$$+ b_1 \sin kt + b_2 \sin 2kt + \ldots$$

if carried out to a sufficient number of terms will reproduce almost any type of graph. This equation may be expressed also in the form

$$y = A_0 + A_1 \sin (kt + e_1) + A_2 \sin (2kt + e_2) + \ldots$$

When the parameters of this equation are determined from statistical data the question arises as to the significance of the several terms in the Fourier series. Do the successive terms in the sine series correspond to real periodicities in nature, or are they merely formal terms the summation of which will give the observed values of the original data? If, for example, a high value were obtained for one of the A-coefficients in the sine series, what warrant would there be for assuming that the particular sine term of which the given A was the

[1] H. H. Turner, Tables for Facilitating the Use of Harmonic Analysis, p. 44.

coefficient would be significant of a real recurring periodicity?

This problem was considered by Professor Schuster in his theory of the periodogram.[1] According to Professor Schuster, "the periodogram may be said to put the statistical material in a form in which it may be most readily discussed, but there may be always cases in which the interpretation is difficult. . . . I do not, of course, claim to have first introduced the application of Fourier's Theorem to the discovery of hidden periodicities. . . . The process is sufficiently obvious to have been frequently introduced, but it has generally been assumed that each maximum in the amplitude of a harmonic term corresponded to a true periodicity. What distinguishes the method which I am endeavouring to introduce from that of others, is the discussion of the natural variability of the Fourier coefficients according to the theory of probability, independently of any periodic cause which may have influenced the phenomenon."[2]

The first step in the Schuster method of isolating true periodicities by the method of the periodogram consists in arranging the data of the statistical series into groups of different lengths and then computing the values of the coefficients of the sine terms appropriate to the different groups. The required lengths and closeness of the groups are discussed[3] by Professor Schuster, and he

[1] The fundamental memoirs of Professor Schuster are

"On the Investigation of Hidden Periodicities with Application to a Supposed 26 Day Period of Meteorological Phenomena." Terrestrial Magnetism, for March, 1898; "The Periodogram of Magnetic Declination as Obtained from the Records of the Greenwich Observatory during the Years 1871–1895," Cambridge Philosophical Society Transactions, vol. 18, 1899; "On the Periodicity of Sunspots," Philosophical Transactions of the Royal Society of London, A, vol. 206, 1906.

[2] "On the Periodicities of Sunspots," Philosophical Transactions, A, vol. 206, pp. 71, 72.

[3] See particularly "On the Periodicities of Sunspots," Philosophical Transactions, A, vol. 206, p. 71.

further shows how the probability of the reality of any assumed cycle is dependent upon the magnitude of the coefficient of its corresponding harmonic in the periodogram. In brief, the probability of the reality of an assumed cycle is shown to be dependent upon the relative size of A^2 where A is the coefficient of the sine term descriptive of the assumed cycle.

Professor Turner's method, which he has called the method of Fourier Sequence,[1] is based upon Professor Schuster's method of the periodogram. The device may be illustrated by the problem in connection with which the method of Fourier Sequence was developed. In 1913, when Professor Turner published his essays, a fairly good record of sunspots existed for a period of 156 years. His problem was to determine whether there was ground for believing that there were real periodicities in the sunspot data, and if so, to ascertain their approximate lengths. The same problem had been considered by Professor Schuster with the aid of the method of the periodogram, but, according to Professor Turner, the Schuster method was needlessly complex, involving an unnecessary amount of computation. An adequate solution was thought to be obtained if a Fourier series were computed for the whole series of data, and the several terms of the series were investigated more in detail according as the magnitudes and signs of the coefficients of the several terms indicated the possible presence of a real cycle. In order to carry out this idea, Professor Turner computed for the 156 years of sunspot data the harmonics for the periods of the following number of

[1] The fundamental memoirs of Professor Turner are "On the Harmonic Analysis of Wolf's Sun-spot Numbers, With Special Reference to Mr. Kimura's Paper," Monthly Notices of the Royal Astronomical Society, May, 1913; "On the Expression of Sun-spot Periodicity as a Fourier Sequence, and on the General Use of a Fourier Sequence in Similar Problems," ibid., 1913 (Supplement); "Further Remarks on the Expression of Sun-spot Periodicity as a Fourier Sequence," ibid., November, 1913.

years: 156, 156/2, 156/3 . . . 156/54. The principal advantage claimed by Professor Turner for the method of Fourier Sequence is this: The development of the Fourier Sequence is

(1) *Necessary.* — "Since each term of the Fourier Sequence is independent of every other, it cannot be inferred from any other. Hence we must at least calculate all these terms." [1]

(2) *Sufficient.* — "If we desire to know to what extent any periodicity intermediate between two of those directly tabulated is represented in the observations . . . we are able to deduce this information from the sequence." [2]

II

An Analysis of a Century of Prices

As a preliminary step our investigation will follow the method of Professor Turner in the analysis of Sauerbeck's index numbers of general wholesale prices.

Sauerbeck's index numbers of general wholesale prices from 1818 to 1913 are recorded in Table I of the Appendix and are graphed in Figure 1. The graph shows quite clearly that the mean value of the items in the early part of the series is higher than the mean value during the latter part, and we are, therefore, confronted with the question as to what shall be done about the secular trend of the figures. Any hypothesis that might be made as to the type of curve to represent the secular trend would, to a degree, be an arbitrary hypothesis, and my decision has been to make no supposition as to the secular trend, but to proceed with the computation of the Fourier terms from the crude index numbers. In

[1] "On the Expression of Sun-spot Periodicity as a Fourier Sequence," Monthly Notices of the Royal Astronomical Society, 1913, p. 715.

[2] Ibid., p. 715.

support of this decision these two considerations are offered:

1. It is known that each term of a Fourier sequence is independent of the other terms, and there is, therefore, a probability that when a Fourier series is fitted to the statistical data covering a considerable length of time, the early terms of the series will make an allowance for the secular trend which will be independent of the later terms.[1] Figures 1, 2, 3, 4 show the reasonableness of this assumption. An ample description of these graphs will be given later on.

2. The authority of Professor Schuster is against the early elimination of the secular trend before the Fourier terms are computed: "Very considerable labour has sometimes been spent in eliminating secular variations and other known periodicities before the hidden periodicities are searched for. We may reasonably ask the question, what object is thereby gained? It is one of the great advantages of Fourier's analysis that each of its terms is independent of the others; and if we wish to determine any particular coefficient it is unnecessary to begin by eliminating the others. The analysis itself performs that process in the best possible way, if the coefficients are obtained by arithmetical calculations. . . . The general rule may be given, that it is the best to eliminate as few variations as possible, and to carry out the elimination at as late a stage of the computation as possible."[2]

For these reasons we have computed the Fourier terms directly from the index numbers in the raw state. The results of the computation are given in Table II

[1] Cf. Schuster, "The Periodogram of Magnetic Declination," p. 113. "Table . . . clearly shows the effects of secular variation, and we must consider in how far it is necessary to take any notice of this variation. If our observations extended over an indefinite time, Fourier's analysis would itself perform all that is required, and each period would be totally independent of all others."

[2] "On the Investigation of Hidden Periodicities," p. 34. Cf. also p. 38.

of the Appendix. The headings of the table will be understood from an examination of Fourier's series when it is expressed in the following two forms:

$$(1)\ y = A_0+a_1 \cos kt+a_2 \cos 2kt+ \ . \ . \ . \\ +b_1 \sin kt+b_2 \sin 2kt+ \ . \ . \ .$$

$$(2)\ y = A_0+A_1 \sin (kt+e_1)+A_2 \sin (2kt+e_2)+ \ . \ . \ .$$

The amplitudes of the terms in (2) — the A-coefficients — are obtained from the corresponding coefficients in (1) by means of the formula $A = \sqrt{a^2+b^2}$. A_0 is equal to the mean value of the 96 index numbers. The lengths of the periods in the second column of Table II are obtained by dividing 96, which is the number of years of observations recorded in the Sauerbeck index numbers, by the consecutive integers that are given in the first column. The constant k in the formulae (1) and (2), which does not appear in Table II, is equal to $\frac{360°}{96} = 3° \ 45'$. The constants e in (2) do not occur in Table II, but their values may be obtained from the corresponding values of a and b in (1) by the relation $\tan e = \frac{a}{b}$.

We shall now consider the conclusions that may be drawn from the data of Table II and we shall begin with the last column which gives the values of A^2. If we let the eye run down the last column, it will note that at four places the values of A^2 assume special importance — for the periods of 96 years, 48 years, 24 to 16 years, and 8.7 to 7.4 years. A moment ago when the question of eliminating the secular trend was under consideration, the decision was reached to permit the early terms of the Fourier series to provide for the secular trend rather than to adjust the raw figures of the observation according to a more or less arbitrary assumption as to the type

of curve which might be appropriate to represent the trend. It now seems reasonable to conclude that the large values of A^2 at 96 years and 48 years, and possibly those between 24 and 16 years, are caused by the general trend of the figures. Inasmuch as one of these covers the whole range of the observations and the other, one-half of the range, one would certainly not be justified in holding that they represent real recurrent cycles. Figures 1 and 2 depict these two Fourier constituents of the price curve. Figure 1 also shows the curve that is obtained by combining the mean of the Sauerbeck index numbers with the 96-year Fourier constituent. Likewise Figure 2 gives the compound curve made up of the mean of the Sauerbeck index numbers and the 96- and 48-year Fourier constituents.

With two of the four important values of A^2 in Table II accounted for, the possibility of real cycles in the 96-years record is limited to the remaining two epochs between 24 and 16 years and between 8.7 and 7.4 years. The mean of the limiting values of the latter period is $\frac{8.7+7.4}{2} = 8.0$ years, and the limits of the other period are respectively twice and three times this mean value. The value of A^2 in Table II corresponding to a period of exactly 8 years is small, but if the computation had been confined to the interval 1857–1913 its value would have been 7.95.

Thus far the argument as to the existence of real periods has been based upon the size of A^2 which is the criterion used by Professor Schuster. An additional criterion has been offered by Professor Turner. In his fundamental memoir [1] he has pointed out that when a striking periodicity is present, there is a tendency for the

[1] H. H. Turner, "On the Expression of Sun-spot Periodicity as a Fourier Sequence," Monthly Notices of the Royal Astronomical Society, 1913, p. 716. Cf. also, pp. 722, 723.

signs of a and b to change between consecutive Fourier constituents. Table II shows that not only are the values of A^2 large for the period between 24 and 16 years, and for the period between 8.7 and 7.4 years, but that in both instances there is a change of sign in either a or b.

Considering that in a record of 96 years a possible cycle of about 16 years could occur six times and one of about 8 years could occur twelve times, this analysis of a century of prices seems to warrant the conclusion that there may be a real cycle of prices between 16 and 24 years in length, and that there is a large probability of the existence of a real cycle in the neighborhood of 8 years. As the argument proceeds we shall have strong additional reason for believing that the indicated eight-year cycle in the Sauerbeck index numbers of wholesale prices is a real cycle with an assignable cause. Certainly the Fourier analysis indicates that if there is a real cycle in the 96 years of the Sauerbeck observations, its most probable value is in the neighborhood of eight years.

In order to isolate and exhibit the cycles of approximately eight years the graphs of Figures 3, 4, 5 have been computed and drawn. Figure 3 shows the Fourier constituent of 19.2 years — which, according to Table II, is one of the most important constituents — and the compound curve that results from combining the mean of the Sauerbeck observations with the Fourier constituents of 96, 48, and 19.2 years. Figure 4 depicts the 16-year Fourier constituent and the compound curve made up of the mean value of the Sauerbeck numbers and the 96-, 48-, 19.2- and 16-year Fourier constituents. A review of Figures 1, 2, 3, 4 exemplifies how the addition of Fourier terms gives an increasingly accurate description of the general trend of the Sauerbeck num-

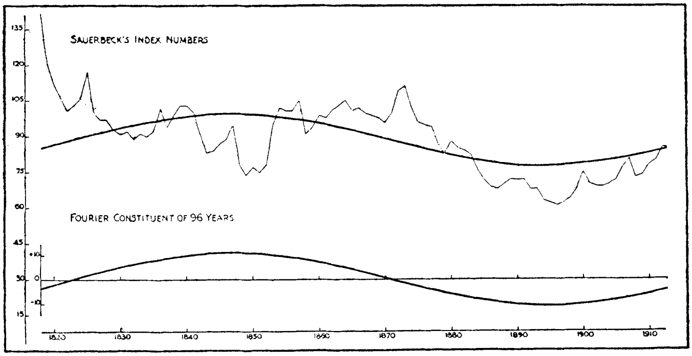

FIGURE 1. Sauerbeck's index numbers of general wholesale prices. Equation to the upper smooth curve: $y = 88.6 + 11.2 \sin (\frac{360}{96}°t + 342°)$, origin at 1818.

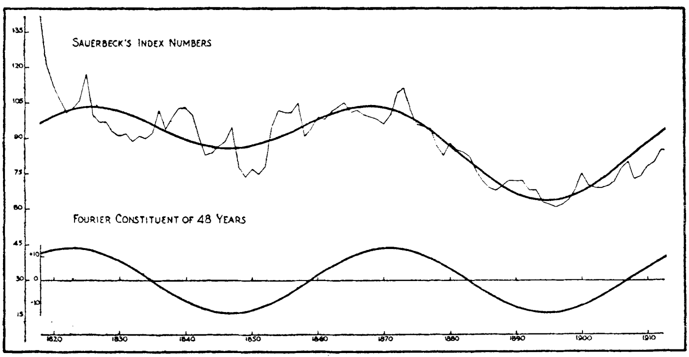

FIGURE 2. Sauerbeck's index numbers of general wholesale prices. Equation to the upper smooth curve: $y = 88.6 + 11.2 \sin (\frac{360}{96}°t + 342°) + 13.8 \sin (\frac{360}{48}°t + 55°)$, origin at 1818.

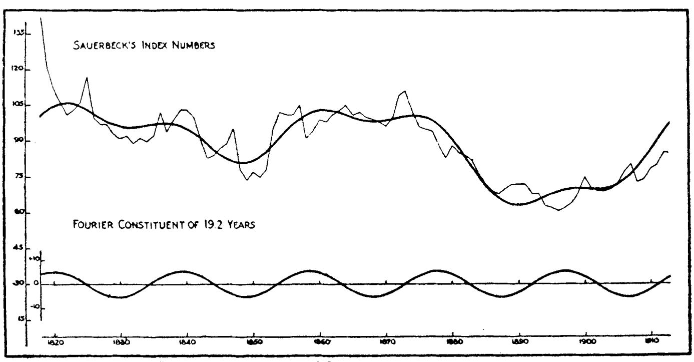

FIGURE 3. Sauerbeck's index numbers of general wholesale prices. Equation to the upper smooth curve: $y = 88.6 + 11.2 \sin (\frac{360}{96}°t + 342°) + 13.8 \sin (\frac{360}{48}°t + 55°) + 5.3 \sin (\frac{360}{19.2}°t + 52°)$, origin at 1818.

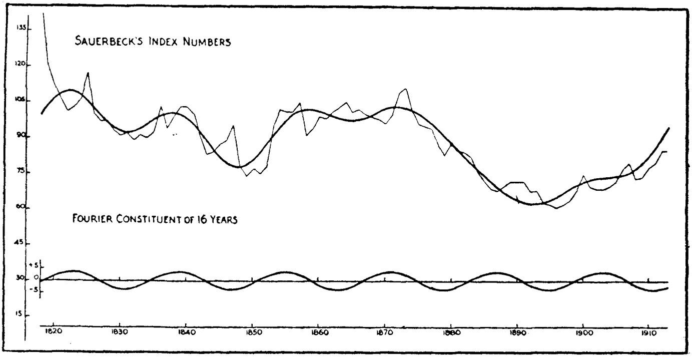

FIGURE 4. Sauerbeck's index numbers of general wholesale prices. Equation to the upper smooth curve: $y = 88.6 + 11.2 \sin (\frac{360^\circ}{96}t + 342^\circ) + 13.8 \sin (\frac{360^\circ}{48}t + 55^\circ) + 5.3 \sin (\frac{360^\circ}{19.2}t + 52^\circ) + 3.6 \sin (\frac{360^\circ}{16}t + 343^\circ)$, origin at 1818.

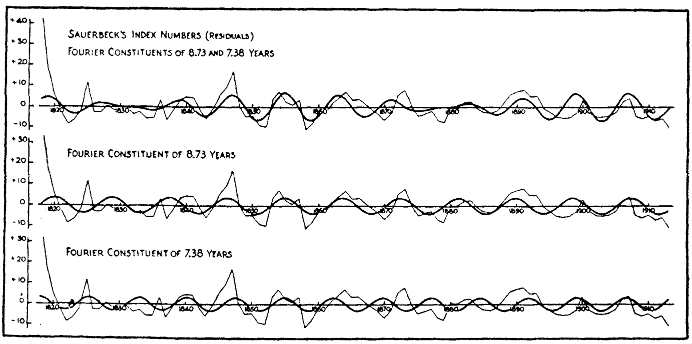

FIGURE 5. Residuals of Sauerbeck's index numbers of general wholesale prices. Equation to the upper smooth curve: $y = 3.8 \sin (\frac{360^\circ}{8.73}t + 9^\circ) + 3.1 \sin (\frac{360^\circ}{7.38}t + 92^\circ)$, origin at 1818.
Middle smooth curve: $y = 3.8 \sin (\frac{360^\circ}{8.73}t + 9^\circ)$.
Bottom smooth curve: $y = 3.1 \sin (\frac{360^\circ}{7.38}t + 92^\circ)$.

bers. The compound curve in Figure 4, I have regarded as the general trend of the Sauerbeck numbers. If the ordinates of this compound curve corresponding to the years between 1818 and 1913 are subtracted from the Sauerbeck index numbers for those years, we obtain what I have called the Sauerbeck residuals, which are listed in Table III and are graphed in Figure 5.

Table II, as we have seen, indicates that there is a real cycle between 8.73 and 7.38 years in the Sauerbeck index numbers. In Figure 5, three smooth curves have been fitted to the Sauerbeck residuals, one of which is made up of the two cycles of 8.73 and 7.38 years respectively. This curve gives an excellent description of the residuals. In the remaining curves the cycles of 8.73 and 7.38 years have been fitted to the data separately. The curve of 8.73 years, as we should expect from the greater value of A^2 in Table II, gives a better description of the residuals than the curve of 7.38 years.

III

Crop Cycles as Generating Cycles

No one familiar with the theory of prices and with their multitudinous causes of change would expect the record of general wholesale prices to show an exact mathematical precision in the working out of any one cause. If there were a predominant cause one would feel that, at best, the nature of its effect would be revealed only in the average of a fairly long record. One would be prepared for a marked deviation from the average in any particular instance. On the other hand, if in the average of a fairly long record there should be evidence of a persistent cycle, no one acquainted with the statistical theory of cycles would fail to suspect the presence of a predominant periodic cause.

The analysis of a century of prices has revealed the existence of a cycle in wholesale prices of about eight years in length. What is its cause?

In the paper [1] to which reference has already been made, I have shown that in the United States the yield per acre of the leading crops between 1882 and 1918

[1] "Generating Cycles of Products and Prices," Quarterly Journal of Economics, February, 1921.

passed through cycles of about eight years with maxima at about 1882, 1890, 1898, 1906, and 1914. These eight-year cycles in the yield of the crops generated eight-year cycles in the prices of organic raw materials of manufactures which, according to the law of competitive price, were followed by corresponding cycles in the prices of manufactured commodities. In two earlier articles published in the *Journal of the Royal Statistical Society* the following were among the conclusions that were reached: [1]

(1) The yield per acre of representative crops in the United Kingdom since 1884 — when the figures for the yield per acre of the crops began to be collected officially—passed through eight-year cycles which were synchronous with the cycles of eight years in the yield of the American crops;

(2) The yield per acre of representative crops in France was closely correlated with the yield in the United Kingdom and passed through eight-year cycles which were synchronous with the crop cycles in the United Kingdom and in the United States.

The synchronism of the crop cycles in these three countries and the demonstrated existence of derived eight-year cycles of agricultural prices in the United States, which according to the law of competitive price induced corresponding cycles in the prices of manufactured commodities, would seem to indicate that the clue to the observed eight-year cycle in Sauerbeck's index numbers of general prices might be found in the eight-year cycles of the crops.

Holding fast to this clue we shall present evidence to

[1] "Crop Cycles in the United Kingdom and in the United States," May, 1919 "Crop Cycles in the United Kingdom and in France," May, 1920.

show that throughout the interval under investigation, 1818 to 1913, and for a still longer period, the British crops passed through cycles of approximately eight years in length.

The first bit of evidence has already been adduced. The yield per acre of representative crops in the United Kingdom — wheat, oats, and barley — passed through cycles of eight years with maxima at about 1882, 1890, 1898, 1906, 1914, and these cycles were synchronous with those of France and the United States. (The graph is given in Figure 6.)

In presenting the next remarkable piece of evidence I make use of a thoughtful, long neglected paper [1] on "A Comparison of the Fluctuations in the Price of Wheat and in the Cotton and Silk Imports into Great Britain," by the late J. H. Poynting, at one time Professor of Physics in Birmingham.[2] Having in mind, doubtless, the essays of Stanley Jevons, the author expressed cautiously the opinion that "the attempt to prove the sunspot origin of variations of the harvests and crops has probably led us somewhat away from the proper line of inquiry. This, it seems to me, should begin with such a manipulation of the statistics as to show the true fluctuations whatever they may be, with the effects of wars, increase of commerce, etc., as far as possible eliminated." [3] Accordingly Professor Poynting set about devising a method to reveal the essential fluctuations in the price of wheat in England from 1760 to 1875. Here is his description of the method:

> In order to determine the fluctuations we require to know not only the actual price, but whether that price is above or below the

[1] Journal of the Royal Statistical Society, March, 1884.

[2] "Poynting belonged to the rare type of men who are more critical of their own work than of that produced by others. The number of his papers is therefore comparatively small, but each of them marks some definite and generally important step." Schuster and Shipley, Britain's Heritage of Science, p. 161.

[3] Journal of the Royal Statistical Society, March, 1884, p. 35.

average *for that time.* It becomes necessary then to average the prices in some way so as to obtain a standard for each year, and we can then determine whether the price for any particular year is high or low according as it is above or below that standard. I have found it sufficient to average for ten years at a time, that is, I have taken as the standard for each year the average of the ten years of which that year is the fifth. If a curve be drawn whose ordinates represent these standard prices, it will be seen at once that all the larger irregularities are nearly smoothed down. . . .

It would now be possible to represent the rises and falls in price by comparing the price for each year with the standard for that year. But there are so many irregularities of short duration, say two or three years, that it is more convenient to take, instead of the price for each year, the average for a short period, and for this purpose I adopt four years. The price for any one year then to be compared with the standard, is the average for the four years of which that year is the second.

Were there only very small variations in the standard, it would be sufficient to take the difference between the ten-yearly and the four-yearly averages. But the standard varies very considerably. . . . The higher the standard, the greater are the differences between it and the four-yearly average. To obtain results which may be compared with each other at different times, this effect of change of standard must be eliminated. This may be done by finding what percentage the four-yearly is of the ten-yearly average.[1]

The numerical results of the application of this method to the prices of wheat in England from 1760 to 1875 are given in Table IV and are graphed in Figure 6. After Professor Poynting had made his computations following the method which has just been described and had written his paper, he had the good fortune to have it read, before its publication, by Professor George Gabriel Stokes [2] than whom there was probably no one

[1] Journal of the Royal Statistical Society, March, 1884, pp. 36, 37.

[2] "The golden age of mathematics and physics at Cambridge was coincident with the scientific activity of George Gabriel Stokes (1819–1903) which began in 1842, and extended, with but slightly diminished vigour, to the end of the last century. Stokes' position as an investigator is among the greatest, but his influence cannot be measured merely by the record of his published work. He united two generations of scientific workers by the love and veneration centered in their gratitude for the assistance and encouragement which, with kindly and genuine interest, he showered upon them out of the wealth of his knowledge and experience. Even those who intellectually were his equals owed much to his sound and impartial judgment. Turning away from the grave which

better equipped to pass judgment upon the mathematical implications of the Poynting method of curve smoothing. Professor Poynting's conclusions from the observations of his eminent critic are given in these summary sentences:

> Thus the effect of the averaging process is practically to destroy all harmonics below five years, to save over half the amplitude at six years, a greater amount up to eight years, when about five-sixths is saved, and beyond that a continually decreasing amount, though at fifteen years still nearly one-half is saved. . . . Thus while for eight, nine, and ten-year periods the process saves about 80 per cent of the coefficient, it falls to 60 per cent on the one side for six years, and to 45 per cent on the other for sixteen years.[1]

With this large proportion of the amplitudes of possible cycles between six and sixteen years in length preserved by the process of curve smoothing, one would suppose that the author would have been eager to know whether there was any regularity in his data. But Professor Poynting's caution led him to stop at the most interesting phase of his investigation. The Chairman of the meeting of the Statistical Society at which the paper was read, Sir Rawson W. Rawson, was at pains specifically to point out that "Professor Poynting did not suggest that there were periods, or cycles of prices or of anything similar. He had merely adopted a ten year's period for forming an average, in order to establish a curve for the examination and comparison of periodicities of every kind, and did not suggest that there were periods, or cycles of prices, or anything else." [2]

We now ask the definite question: Is there evidence that cycles in the yield of wheat recurred in the interval

was closed over his life-long friend, Kelvin was heard to say: 'Stokes is gone, and I shall never return to Cambridge.'" Schuster and Shipley, Britain's Heritage of Science, p. 123.

[1] Journal of the Royal Statistical Society, March, 1884, pp. 46, 47.

[2] Ibid., p. 68.

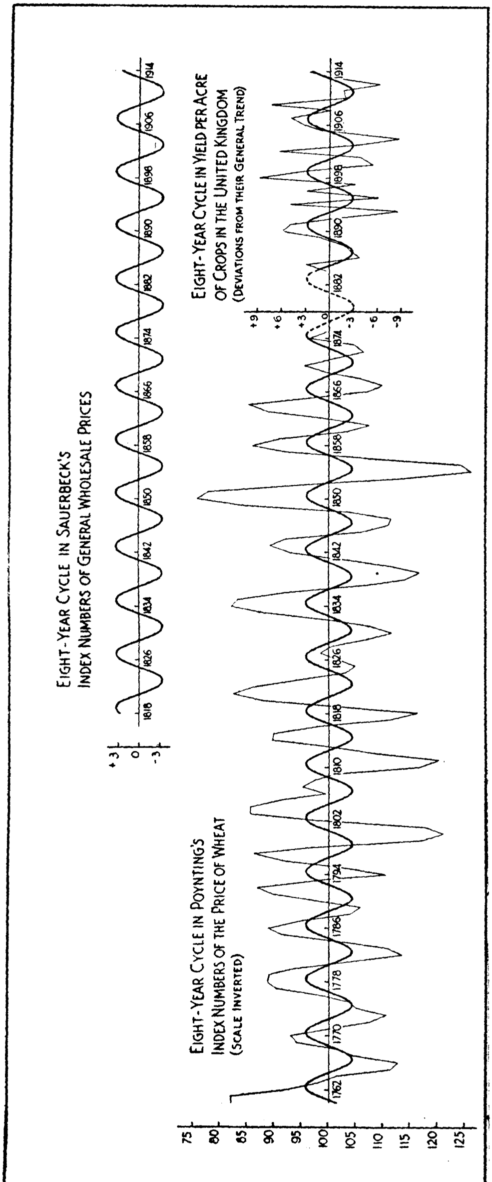

FIGURE 6. Generating cycles reflected in a century of prices.

Upper curve: $y = 3.5 \sin\left(\frac{360°}{8}t + 51°\right)$, origin at 1818. Derived from the 8.73 and 7.38 years Fourier constituents of Sauerbeck's index numbers.

Lower curves: Poynting's index numbers, $y = 100.2 + 4.4 \sin\left(\frac{360°}{8}t + 248°\right)$, origin at 1762;

Yield per acre of the crops, $y = 3.0 \sin\left(\frac{360°}{8}t + 142°\right)$, origin at 1884.

from 1760 to 1875? By the application of the method of the periodogram we have found that in the yield per acre of the leading crops in the United States, in the United Kingdom, and in France, there are cycles of approximately eight years, and these eight-year cycles in the three countries are synchronous. We have also found that the prices of the crops in the United States are closely correlated with the yield per acre, the coefficients of correlation [1] ranging from $r = -.2$ to $r = -.9$ and averaging $r = -.7$.

It would, therefore, seem legitimate to assume that if there were cycles in the price of wheat in England from 1760 to 1875 during a large part of which time the importation of grain was restricted in consequence either of wars or of corn laws, then there were cycles of like period and opposite phase in the yield per acre of the crops.

If we compute a sine curve with a period of eight years for the Poynting smoothed index numbers of the price of wheat, which are given in Table IV, we find (Figure 6) that

(1) The curve fits the observations surprisingly well throughout the 116 years of the record except for the interval of the wars of the French Revolution and the Napoleonic wars;

(2) The eight-year cycle in the price of wheat, in consequence of the law of demand, reveals an eight-year cycle in the yield per acre of wheat continuous with the eight-year cycle already established for the crops of the United Kingdom [2] from 1884 to 1914. The continuity of the cycles is shown in Figure 6.

(3) The cycles in the yield per acre of the crops

[1] "Generating Cycles of Products and Prices," Quarterly Journal of Economics, February, 1921, pp. 223, 226.

[2] The data and original graph for the period 1884 to 1914 may be found in the Journal of the Royal Statistical Society, May, 1919, pp. 384, 387.

from 1760 to 1913 generated derived cycles in the prices of organic raw materials of production which, in consequence of the law of competitive price, must have tended to induce eight-year cycles in the general prices of commodities;

(4) The analysis of the Sauerbeck index numbers shows that, as a matter of fact, during the century for which the Sauerbeck numbers are given, 1818–1913, general prices did pass through cycles of approximately eight years.

In Figure 6 an eight-year cycle is fitted to the residuals of the Sauerbeck index numbers. We found in our analysis of the Sauerbeck residuals that the equation to the indicated cycle of 8.73 years was

$$y = 3.8 \sin \left(\frac{360^\circ}{8.73} t + 9^\circ\right),$$

and that the equation to the indicated cycle of 7.38 years was $y = 3.1 \sin \left(\frac{360^\circ}{7.38} t + 92^\circ\right)$. The eight-year cycle which is representative of the Sauerbeck residuals in Figure 6 was constructed from these two equations in the following way: Its period of eight years is the mean of the periods of these two cycles $\frac{8.73+7.38}{2} = 8.05$; the amplitude of the eight-year cycle is the mean of the amplitudes of these two cycles, $\frac{3.8+3.1}{2} = 3.45$; and the phase of the eight-year cycle is the mean of the phases of the two cycles $\frac{9^\circ+92^\circ}{2} = 50^\circ\,30'$. It will be seen from Figure 6 that the eight-year cycles in the cen-

tury of Sauerbeck's index numbers were approximately synchronous with the corresponding cycles in the indicated yield per acre of the British crops. The eight-year cycle in the crops is proved to have persisted throughout nearly the whole period of 159 years from 1760 to 1918 (the investigation of the American crops was carried through 1918) or for an interval of twenty cycles of eight years in length. This generating eight-year cycle in the crops induced derived cycles of prices which are reflected and verified in the century of Sauerbeck index numbers of general wholesale prices.

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APPENDIX

TABLE I. — SAUERBECK'S INDEX NUMBERS OF GENERAL WHOLESALE PRICES

Year	Index Number	Year	Index Number	Year	Index Number	Year	Index Number
1818	142	1842	91	1866	102	1890	72
1819	121	1843	83	1867	100	1891	72
1820	112	1844	84	1868	99	1892	68
1821	106	1845	87	1869	98	1893	68
1822	101	1846	89	1870	96	1894	63
1823	103	1847	95	1871	100	1895	62
1824	106	1848	78	1872	109	1896	61
1825	117	1849	74	1873	111	1897	62
1826	100	1850	77	1874	102	1898	64
1827	97	1851	75	1875	96	1899	68
1828	97	1852	78	1876	95	1900	75
1829	93	1853	95	1877	94	1901	70
1830	91	1854	102	1878	87	1902	69
1831	92	1855	101	1879	83	1903	69
1832	89	1856	101	1880	88	1904	70
1833	91	1857	105	1881	85	1905	72
1834	90	1858	91	1882	84	1906	77
1835	92	1859	94	1883	82	1907	80
1836	102	1860	99	1884	76	1908	73
1837	94	1861	98	1885	72	1909	74
1838	99	1862	101	1886	69	1910	78
1839	103	1863	103	1887	68	1911	80
1840	103	1864	105	1888	70	1912	85
1841	100	1865	101	1889	72	1913	85

TABLE II. — RESULTS OF FOURIER ANALYSIS OF SAUERBECK'S INDEX NUMBERS OF GENERAL WHOLESALE PRICES. 1818–1913

Divisor	Period in Years	a	b	A^2	Divisor	Period in Years	a	b	A^2
1	96.0	− 3.46	+10.60	124.42	13	7.4	+3.12	− .11	9.74
2	48.0	+11.27	+ 7.96	190.40	14	6.9	+2.12	+1.77	7.62
3	32.0	+ 1.08	+ .76	1.73	15	6.4	+ .61	+ .17	.41
4	24.0	+ 2.95	+ .70	9.17	16	6.0	+1.67	+ .29	2.86
5	19.2	+ 4.18	+ 3.28	28.23	17	5.6	+ .88	+ .72	1.30
6	16.0	− 1.08	+ 3.46	13.13	18	5.3	+ .59	+2.03	4.48
7	13.7	+ .78	+ .76	1.19	19	5.0	+ .04	+ .66	.44
8	12.0	+ 1.85	+ .73	3.95	20	4.8	+ .29	+ .75	.65
9	10.7	+ 1.18	− .02	1.40	21	4.6	+1.38	+ .48	2.14
10	9.6	+ 2.00	− .92	4.86	22	4.4	+ .03	+1.64	2.68
11	8.7	+ .61	+ 3.76	14.52	23	4.2	+ .80	+ .63	1.03
12	8.0	+ 1.36	− .51	2.12	24	4.0	+ .30	− .10	.10

TABLE III. — RESIDUALS OF SAUERBECK'S INDEX NUMBERS OF GENERAL WHOLESALE PRICES

Year	Residual	Year	Residual	Year	Residual	Year	Residual
1818	+42.4	1842	− 1.2	1866	+ 3.9	1890	+ 7.7
1819	+17.7	1843	− 5.7	1867	+ 1.2	1891	+ 8.5
1820	+ 5.5	1844	− 0.9	1868	− 1.0	1892	+ 5.2
1821	− 2.6	1845	+ 5.3	1869	− 3.2	1893	+ 5.2
1822	− 8.4	1846	+ 9.7	1870	− 6.6	1894	− 0.5
1823	− 6.3	1847	+17.0	1871	− 2.8	1895	− 2.4
1824	− 1.7	1848	+ 0.2	1872	+ 5.9	1896	− 4.7
1825	+11.6	1849	− 4.8	1873	+ 8.2	1897	− 5.3
1826	− 2.4	1850	− 4.1	1874	+ 0.3	1898	− 5.0
1827	− 2.4	1851	− 9.0	1875	− 4.3	1899	− 2.5
1828	+ 0.4	1852	− 9.5	1876	− 3.3	1900	+ 3.2
1829	− 1.3	1853	+ 3.7	1877	− 1.9	1901	− 2.9
1830	− 1.8	1854	+ 7.3	1878	− 6.3	1902	− 4.5
1831	− 0.4	1855	+ 2.5	1879	− 7.5	1903	− 4.7
1832	− 3.6	1856	+ 1.0	1880	+ 0.5	1904	− 4.1
1833	− 2.8	1857	+ 3.6	1881	+ 0.3	1905	− 2.4
1834	− 5.6	1858	−10.9	1882	+ 2.0	1906	+ 2.1
1835	− 5.3	1859	− 7.9	1883	+ 2.7	1907	+ 4.1
1836	+ 3.0	1860	− 2.1	1884	− 0.7	1908	− 4.4
1837	− 6.2	1861	− 2.2	1885	− 2.4	1909	− 5.8
1838	− 1.5	1862	+ 1.8	1886	− 2.9	1910	− 4.8
1839	+ 3.2	1863	+ 4.8	1887	− 1.7	1911	− 6.5
1840	+ 4.9	1864	+ 7.2	1888	+ 2.3	1912	− 5.7
1841	+ 4.5	1865	+ 3.3	1889	+ 6.1	1913	−10.2

TABLE IV. — POYNTING INDEX NUMBERS OF THE PRICE OF WHEAT PER QUARTER IN ENGLAND FROM 1760 TO 1875.
M_{10} = TEN-YEAR AVERAGE OF THE ABSOLUTE PRICES OF WHEAT
M_4 = FOUR-YEAR AVERAGE OF THE ABSOLUTE PRICES OF WHEAT

Year	Poynting Index M_4/M_{10}	Year	Poynting Index M_4/M_{10}	Year	Poynting Index M_4/M_{10}	Year	Poynting Index M_4/M_{10}
1760	82.3	1789	106.0	1818	116.6	1847	111.6
1761	82.2	1790	98.5	1819	103.5	1848	103.6
1762	92.1	1791	90.0	1820	87.9	1849	84.2
1763	98.1	1792	87.1	1821	82.6	1850	75.9
1764	101.8	1793	98.1	1822	86.2	1851	78.0
1765	111.6	1794	110.7	1823	95.0	1852	94.8
1766	112.9	1795	102.3	1824	103.3	1853	112.9
1767	104.8	1796	91.9	1825	105.0	1854	126.4
1768	101.4	1797	86.4	1826	101.7	1855	124.6
1769	94.3	1798	97.0	1827	98.6	1856	108.5
1770	93.3	1799	117.4	1828	100.9	1857	92.2
1771	100.2	1800	121.1	1829	107.0	1858	86.8
1772	107.7	1801	117.8	1830	111.7	1859	91.4
1773	110.7	1802	98.4	1831	107.8	1860	103.0
1774	105.3	1803	85.7	1832	100.2	1861	107.7
1775	103.8	1804	85.7	1833	87.6	1862	99.2
1776	98.3	1805	91.3	1834	82.3	1863	88.8
1777	90.5	1806	99.7	1835	83.3	1864	85.3
1778	88.9	1807	95.4	1836	91.9	1865	96.2
1779	89.2	1808	97.6	1837	105.8	1866	107.2
1780	93.9	1809	101.7	1838	114.2	1867	110.0
1781	104.8	1810	116.8	1839	117.0	1868	105.8
1782	113.6	1811	120.3	1840	111.4	1869	99.3
1783	111.0	1812	108.8	1841	101.5	1870	95.5
1784	101.7	1813	100.3	1842	92.9	1871	101.0
1785	91.6	1814	89.8	1843	89.4	1872	106.5
1786	89.1	1815	90.0	1844	92.4	1873	104.7
1787	95.3	1816	97.8	1845	106.1	1874	96.4
1788	104.2	1817	111.3	1846	111.1	1875	99.8